Place in Space – Lesson Plan
Constructing Ellipses and Calculating Eccentricity

Location of "Place in Space"

Gateway to Space

Overview of "Place in Space"

Place in Space is an interactive game found on a stand-alone kiosk. It introduces the Hohmann Transfer Method which allows spacecraft to move from one circular orbit to another (in the same plane). The Hohmann Transfer is the most fuel efficient way to do this. It is accomplished by using an elliptical orbit to transfer between the two circular orbits. It may also be used to maneuver a vehicle in space so it can rendezvous with another vehicle, moon, or planet. (It can also be used to dodge debris.)

The first game level presents a race track with two different exit ramps. One exit goes straight off the track. The other turns at a right angle. The player is asked to choose which can be done without slowing down.

The second game level presents the Earth with a space plane orbiting at a low earth orbit (LEO) which is circular. The player is told that their goal is to go to a medium earth orbit (MEO). To do so, they will thrust their engine to enter into an elliptical orbit. Then, they will thrust again to reach the higher circular orbit. This use of the elliptical orbit is called the Hohmann Transfer. Like with the race car example, it uses the least amount of fuel.

Finally, the player is challenged to complete the Hohmann Transfer to rendezvous their space plane (which is orbiting at a LEO) with a satellite at a higher circular orbit. To earn their badge, they must successfully fire their thruster at the appropriate time.

Lesson Overview

This lesson plan/unit could be used independently or to prepare for (or to extend the learning after) a visit to Spaceport America. It can be used by teachers or parents wishing to make the Spaceport visit a richer learning experience (for everyone!).

Most students will know and recognize a circle. An ellipse, however, may be a new term. (All ellipses are ovals, but mathematically speaking, not all ovals are ellipses.) This lesson is meant to be an introduction to the Hohmann Transfer by requiring students to construct and measure circles and ellipses. Hands-on construction activities will help them to incorporate and integrate the differences between a circle and an ellipse. Calculating eccentricity will help them begin to draw inferences about ellipses. All of this knowledge prepares them for understanding the verbiage used in the Hohmann Transfer. Older students with higher level math skills can use the resource links at the end of this document to learn to calculate the times and thrusts necessary to successfully complete a Hohmann Transfer.

First, a little background and the basics of the Hohmann Transfer Method. For rocket scientists, conserving fuel is very important. In 1925, (even before there were man-made satellites!), a German engineer named Walter Hohmann designed the most fuel efficient way to move a satellite from its initial (circular) parking orbit, to an intermediate (elliptical) orbit, and then to its final (circular) orbit by thrusting tangential to the circular orbit.

To better understand this idea, ask your students to imagine a race car driver traveling around a circular track at a constant *velocity* (speed and direction). To leave the track, ask them if it would it be easier for the driver to take a *turn off* the track or simply *go straight*?

The answer is to exit his “orbit” most efficiently and leave the track, the driver needs only to continue going “straight off the circle” (exit *tangentially*) and simply *slow down (*or *speed up)*. He does not need to change *direction* of travel.

The Hohmann Transfer works just like the race car example mentioned above. Velocity has both *magnitude* (or speed) and *direction*. *Changes in velocity* (or delta Vs) must be *aimed* so that they are tangential to the orbit. That means they must come “straight off” the circle.

To change velocity tangentially, you must fire the thrusters:

- *with* the direction of travel to increase velocity (to go to a higher orbit).
- *opposite* the direction of travel to decrease velocity (to go to a lower orbit).

For rocket scientists, saving energy means saving fuel. Fuel efficiency is always a priority. The Hohmann Transfer method uses tangential exit-ramps and on-ramps to move from one circular orbit to another via an elliptical orbit. The calculations are based on *instantaneous* velocity changes. In reality, the bursts of the engines actually take time. Extra fuel use is minimized by using high burst engines. (Low burst engines must gradually increase velocity and take up to 141% more velocity changes. They also take longer to complete the transfer.)

By introducing the characteristics of ellipses, this lesson can help students see both the value of applied mathematics and think of it as interesting and relevant to everyday life.

Grade Level

Grades 3-8

Learning Objectives

1. Students will be able to construct and interpret circular and elliptical diagrams.
2. Students will be able to explain how ellipses are different from circles and how the eccentricity effects the shape of the ellipse.

Assessment

The student will complete a data table and answer pertinent questions to demonstrate their learning.

Required Materials

For each group you will need:

- Piece of paper
- Cardboard (at least as large as the paper)
- 15 cm string tied into a loop (Don't use yarn because it stretches too much.)
- Two thumbtacks
- Metric ruler
- Calculator (for younger students to use or for all to check their work)

Time Required

One class period

Step-By-Step Procedures:

1. Create a worksheet or duplicate the data table and questions (found in the next section) on the board and ask students to copy.
2. For each piece of string, tie the ends together to form a loop. Try to use up as little string as possible in the knot itself so that the loop is as big as possible.
3. Divide the classroom in to small groups or pairs and distribute the supplies to each group.
4. Ask the students to put one dot in the middle of the paper. Then put the paper on top of the cardboard, and put one thumbtack through the dot. Put the string under the thumbtack, and pulling the string tight, insert the pencil inside the loop and draw a circle. Point out that the circle has just one center point and it is called "the center."
5. Ask the students to remove the paper and fold it into thirds (like you might do when folding a letter to be inserted into an envelope). Make sure they are holding the paper in a portrait orientation and are looking at the *opposite* side of the paper from where they drew the circle. In the top third, tell them to make two dots 2 cm apart. In the middle third, tell them to make two dots 3 cm apart. In the bottom third, tell them to make two dots 4 cm apart.
6. Tell them to put the paper on top of the cardboard once again, and put the two thumbtacks on the dots in the top third. Then tell them to put the loop under both thumbtacks and stretch the loop out with the pencil as far as it will go. Then direct them to move the pencil, continuing to keep the loop taut, and draw the first ellipse.
7. Complete step 6 to draw the other two ellipses.

8. Explain that the dots are each called a *focus*. (The plural is *foci*.) Explain that the *length of the major axis* is the longest diameter across an ellipse. Explain that because an ellipse is *not* a circle it is said to be "eccentric." To calculate the eccentricity of the ellipse, you must divide the distance between the foci by the length of the major axis.
9. Then tell them to fill out their data table and answer the questions.
10. Discuss their data tables and answers to the questions. Answer any other questions they may have.

Worksheet

Calculating Eccentricity

Ellipse	Foci distance (mm)	Length of Major Axis (mm)	Calculate the Eccentricity (foci distance / length of major axis)
Top			
Middle			
Bottom			

A. If you were drawing an ellipse, what would happen to its shape if you used the same size loop but moved the foci even farther apart?

B. The eccentricity of an ellipse can be expressed as a number. If the shape of an ellipse is more nearly round, does its eccentricity increase, decrease, or remain the same?

C. Based on the data you collected, what is the relationship between the eccentricity of an ellipse and how nearly round the ellipse appears to be?

Alignment to Common Core Standards

Links to the Common Core *Geometry* Standards that may apply to your students are below. Choose the standards you feel are most appropriate for your students.

Grades K-8

Resources to Use with Older Students

http://www.stemfortheclassroom.com/2013/01/hohmann-transfer-orbit-equation.html

This link will take you to a lesson plan for pre-calculus high school students. It requires the student to calculate the change in orbital velocity needed to increase (or decrease) the orbital altitude of a spacecraft, calculate the total round-trip time to transfer between orbits, and use that information to determine the duration of the space mission. It is the first in a series of four interconnected astronautics-based S.T.E.M. projects.

http://jwilson.coe.uga.edu/EMT668/EMT668.Student.Folders/BrombacherAarnout/OrbitsLesson/orbits_lesson_series.html

This link will take you to a good lesson plan for even more advanced math students. While some of the internal links are broken, the information presented here and the lesson itself is excellent.